APPENDIX

Equations used in discard estimation and sample size analysis

Total discarded pounds for species *j* is defined as:

$$(1) \quad \hat{D}_j = \sum_{h=1}^{Q} K_h r_{c,j}$$

where

$$(2) \quad r_{c,j} = \frac{\sum_{h=1}^{Q} N_h \sum_{i=1}^{n_h} \frac{d_{jih}}{n_h}}{\sum_{h=1}^{Q} N_h \sum_{i=1}^{n_h} \frac{k_{ih}}{n_h}}$$

Where $\hat{D}_j$ is total discarded pounds for species *j*; K_h is VTR total kept pounds in stratum *h*; $r_{c,j}$ is the combined ratio of species *j*; d_{jih} is discards of species *j* from trip *i* in stratum *h*; k_{ih} is kept pounds of all species on trip *i* in stratum *h*; N_h is the number of VTR trips in stratum *h*; n_h is the number of observed trips in stratum *h*. In Eq. 2 the summation over strata *h* = 1 to *Q* is over calendar quarters and the other strata values are held constant. Equation 3 (below) requires a more explicit definition of the stratum designation since the summation over quarter relies on an annual average ratio defined in Eq. 2.

Variance of $\hat{D}_j$ for species *j* is defined as:

$$(3) \quad V(\hat{D}_j) = \sum_{q=1}^{4} K_{qh}^2 \left(\frac{N_{qh} - n_{qh}}{n_{qh} N_{qh}} \right) \frac{1}{\left(\frac{\sum_{i=1}^{n_h} k_{iqh}}{n_{qh}} \right)^2} \left[\frac{\sum_{i=1}^{n_{qh}} \left(d_{jiqh}^2 + \left(r_{c,j} \right)^2 k_{iqh}^2 - 2 r_{c,j}\, d_{jiqh} k_{iqh} \right)}{n_{qh} - 1} \right]$$

where $\hat{D}_j$ is total discarded pounds for species *j*; K_{qh} is VTR total kept pounds in quarter *q* and stratum *h*; $r_{c,j}$ is the combined ratio of species *j* ; d_{jiqh} is discards of species *j* from trip *i* in quarter *q* and stratum *h*; k_{iqh} is kept pounds of all species on trip *i* in quarter *q* and stratum *h*; N_{qh} is the number of VTR trips in quarter *q* and stratum *h*; n_{qh} is the number of observed trips in quarter *q* and stratum *h*.

Coefficient of variation (CV) of $\hat{D}_j$ is defined as:

(4) $$CV(\hat{D}_j) = \frac{\sqrt{V(\hat{D}_j)}}{\hat{D}_j}$$

The number of sea days and trips needed to achieve a 30% CV are derived based on the variance of the total discards using the combined ratio method and the d/k discard ratio (Eq. 3).

From Eq. 3, let

(5) $$\hat{S}^2_{jqh} = \left[\frac{\sum_{i=1}^{n_{qh}} \left(d^2_{jiqh} + \left(r_{c,jh}\right)^2 k^2_{iqh} - 2 r_{c,j}\, d_{jiqh} k_{iqh} \right)}{n_{qh} - 1} \right]$$ and

(6) $$\delta_{qh} = \frac{n_{qh}}{\sum_{q=1}^{4} n_{qh}}$$

where δ_{qh} is the fraction of the trips in quarter *q* in stratum *h*; $r_{c,jh}$ is the combined annual ratio of species *j* in stratum *h*; d_{jiqh} is discards of species *j* from trip *i* in stratum *h* in quarter *q*; k_{iqh} is kept pounds of all species on trip *i* in stratum *h* in quarter *q*; and n_{qh} is the number of observed trips in stratum *h* in quarter *q*. The $r_{c,jh}$ in Eq. 5 is defined in Eq. 2 where the summation is over quarters within a given strata defined by gear, region, access area, trip type and so forth.

The number of trips necessary to achieve a 30% CV based on the variance of the composite annual total discards for species group *j* in stratum *h* is defined as

(7) $$\hat{T}D_{30\,jh} = \frac{\sum_{q=1}^{4} \left(\frac{K^2_{qh}}{\bar{k}^2_{qh}} \hat{S}^2_{jqh} \frac{1}{\delta_{qh}} \right)}{(0.09)\hat{D}_{jh}{}^2 + \frac{\sum_{q=1}^{4} \frac{K^2_{qh}}{\bar{k}^2_{qh}} \hat{S}^2_{jqh}}{N_h}}$$

where $0.09 = 0.30^2$, the square of the 30%CV, the given target precision level.

The number of sea days necessary to achieve a 30% CV based on the variance of the composite annual total discards for species group *j* in stratum *h* is defined as

(8) $$\hat{S}D_{30\,jh} = \hat{T}D_{30\,jh} * \overline{DA_h}$$

where $\overline{DA_h}$ is the weighted average trip length of VTR trips in stratum *h* (weighted by the number of VTR trips in each quarter) .

When total discards could not be estimated due to little or no observer coverage (no data) or when total discards are zero (no variance), sample size was determined by pilot cover, where 2% of the quarterly VTR trips for a fleet were multiplied by the quarterly mean VTR trip length.

(9) $\hat{S}_{30,jhq} = \hat{T}_{hq} * \overline{DA_{hq}}$

where $\hat{T}_{hq}$ is 2% of the VTR trips in stratum *h* and quarter *q*, and 3 <= $\hat{T}_{hq}$ <= 100 trips; $\overline{DA}_{hq}$ is the average trip length of VTR trips in stratum *h* and quarter *q*. The quarterly trips and sea days were then summed for annual number of trips and sea days.

The achieved precision resulting from the number of funded sea days can be derived by converting funded sea days into funded trips. The number of funded trips, $\hat{TF}_h$ for stratum *h* is defined as:

(10) $\hat{TF}_h = \hat{SF}_h / \overline{DA_h}$

where $\hat{SF}_h$ is the number of funded sea days in stratum *h* and $\overline{DA}_h$ is the weighted average trip length of VTR trips in stratum *h* (weighted by the number of VTR trips in each quarter).

The achieved coefficient of variation (CV) of $\hat{D}_j$ is based on the variance of the composite annual total discards for species group *j* in stratum *h* and the number of funded trips in stratum *h* and re-writing Eq. 7.

From Eq. 7, let

(11) $$CV(\hat{D}_{jh}) = \sqrt{\frac{\sum_{q=1}^{4}\left(\frac{K_{qh}^2}{\bar{k}_{qh}^2}\hat{S}_{jqh}^2\frac{1}{\delta_{qh}}\right) - \hat{TF}_h\left[\frac{\sum_{q=1}^{4}\left(\frac{K_{qh}^2}{\bar{k}_{qh}^2}\hat{S}_{jqh}^2\right)}{N_h}\right]}{\hat{TF}_h * \hat{D}_{jh}^{\,2}}}$$